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A Variable-Order Fractional Total Variation Operator for Multiplicative Noise Removal

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Abstract

Multiplicative noise, commonly referred to as speckle noise, is a major source of degradation in coherent imaging systems such as ultrasound and synthetic aperture radar. Classical total variation regularization and fixed-order fractional models often exhibit limited adaptability, leading to either oversmoothing of fine structures or insufficient noise suppression. In this work, we introduce a variational image restoration model based on a variable-order fractional total variation operator, in which the fractional differentiation order varies spatially according to local image characteristics. The proposed operator is embedded within a rigorous variational framework and combined with a data fidelity term consistent with the statistical nature of multiplicative noise. We analyze the resulting energy functional and establish fundamental theoretical properties, including coercivity, weak lower semicontinuity, and the existence of minimizers. To solve the associated optimization problem efficiently, an operator-splitting algorithm is developed. Numerical experiments on synthetic and real speckled images demonstrate that the proposed variable-order model achieves improved edge preservation and texture fidelity compared to classical total variation and fixed-order fractional approaches, while maintaining effective noise reduction. These results confirm the relevance of variable-order fractional operators for variational inverse problems involving spatially heterogeneous structures.

Keywords: Variable-order operators; fractional total variation; multiplicative noise; speckle noise; variational methods; operator splitting.